

Simulation and Experimentation of the Phase Shifter in High Frequency Pulse Tube Cryocooler

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Abstract. As the demand for compact pulse tube cryocoolers in space applications continues to grow, the lightweight design has become crucial. The phase shifter occupies a significant portion of the system's weight. To optimize this issue, a Sage model was established to investigate the impact of different combinations of inertance tubes on the performance of cryocoolers. An experimental system was set up to validate the simulation results. The experimental data show excellent agreement with simulations and indicate that using a single-stage inertance tube can reduce the weight of the cryocooler with minimal performance compromise.

1. Introduction

The Pulse Tube Cryocoolers (PTCs) have advantages such as low vibration, high reliability, and long lifetime due to the absence of moving components at the cold end, which is widely used in space applications for cooling the infrared detectors. The PTCs adjust the phase relationship between the pressure wave and mass flow at the hot end of the pulse tube through the phase shifter, which affects the overall phase distribution of the system. The optimal phase relationship minimizes losses in the regenerator and maximizes the efficiency of the cryocoolers. The inertance tubes and reservoir are the most widely used phase adjustment methods in PTCs owing to their simple structure and broad adjustment range.

The first reported use of an inertance tube in a PTC was by Kanao in 1994^[1]. In 1997, Zhu et al.^[2] and Gardner and Swift^[3] concluded that an inertance tube produces a larger phase shift than an orifice. Thus, the inertance tube was applied as a new phase shifter in PTCs. For the phase shifter, the primary focus is on the amplitude of the pressure wave and mass flow at the inlet of the inertance tube, as well as the phase angle between them. Radebaugh^[4] pointed out that the efficiency of the PTCs is maximized when the pressure and mass flow are in phase at the midpoint of the regenerator, which requires that the mass flow at the warm end of the pulse tube lag the pressure by about 60 degrees.

Increasing the operating frequency is an effective way to reduce the size of PTCs^[5]. However, it simultaneously exacerbates the challenge of phase adjustment within the inertance tube, as the phase angle along the tube varies more rapidly under high-frequency conditions, making precise phase adjustment considerably more difficult to achieve^[6]. To analyze the impact of different inertance tube combinations on the performance of the micro pulse tube cryocoolers operating at high frequency, a PTC model was developed using Sage software. Based on the simulation



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results, experiments were conducted. This paper aims to provide insights into the selection of inertance tubes in micro PTCs operating at high frequency.

2. Theoretical analysis

The inertance tube exploits the inertial effects of the working fluid to adjust the phase relationship between the pressure wave and the volume flow. By employing an equivalent circuit analogy, the pressure wave can be treated as voltage, while the volume flow is analogous to electric current. The impedance of the fluid system is defined as^[4]

$$Z = \frac{P}{\dot{V}} = \rho_0 \frac{P}{\dot{m}} = \rho_0 Z_m \quad (1)$$

where P is the pressure, \dot{V} is the volume flow rate, and Z_m is the impedance to mass flow. For the fluid transmission line, the inertance, resistance, and compliance per unit length appropriate to Z_m are:

$$l(D) = \frac{4}{\pi D^2} \quad (2)$$

$$r(D) = \frac{2 \cdot 8 f_r |\dot{m}|}{\pi \rho \pi^2 D^5} \quad (3)$$

$$c(D) = \frac{\pi D^2}{4 \gamma R T} \quad (4)$$

where D is the inertance tube diameter, f_r is the Darcy friction factor, γ is the ratio of specific heats, R is the gas constant per unit mass, and T is the average temperature. The complex impedance of a terminated transmission line of length L is given as

$$Z(D, x) = Z_0(D) \frac{Z_r + Z_0(D) \tanh[\varepsilon(D)(L - x)]}{Z_0(D) + Z_r \tanh[\varepsilon(D)(L - x)]} \quad (5)$$

where Z_0 , Z_r , and $\varepsilon(D)$ can be written as (6), (7), and (8), respectively.

$$Z_0(D) = \sqrt{\frac{r(D) + i \omega l(D)}{i \omega c(D)}} \quad (6)$$

$$Z_r = \frac{V_r}{\gamma R T_r} \quad (7)$$

$$\varepsilon(D) = \sqrt{[r(D) + i \omega l(D)] i \omega c(D)} \quad (8)$$

Defining L as the total length of the inertance tube, with $x=0$ at the inlet. The impedance and phase at the inlet of the inertance tube can be determined by solving for $x=0$ in equation(5).

3. Simulation

3.1 Sage model

Sage, a commercial software developed by Gedeon, is a comprehensive thermodynamic analysis tool that integrates modeling, design, and optimization^[7]. It utilizes the finite difference method to solve the mass, momentum, and energy conservation equations. Using this software, a model of the pulse tube cryocooler is established, including three primary components: the compressor, cold finger, and phase shifter, as shown in Fig. 1. Each component consists of several sub-components. In the 'Phase shifter' component, the number of tubes can be adjusted to represent

different combinations of single or multi-diameter inertance tubes. By setting the length of the inertance tubes as the optimization variable, while maintaining constant structural parameters for the compressor and the cold finger, as well as the operating parameters of the cryocooler, the optimal inertance tube length for each combination can be determined.

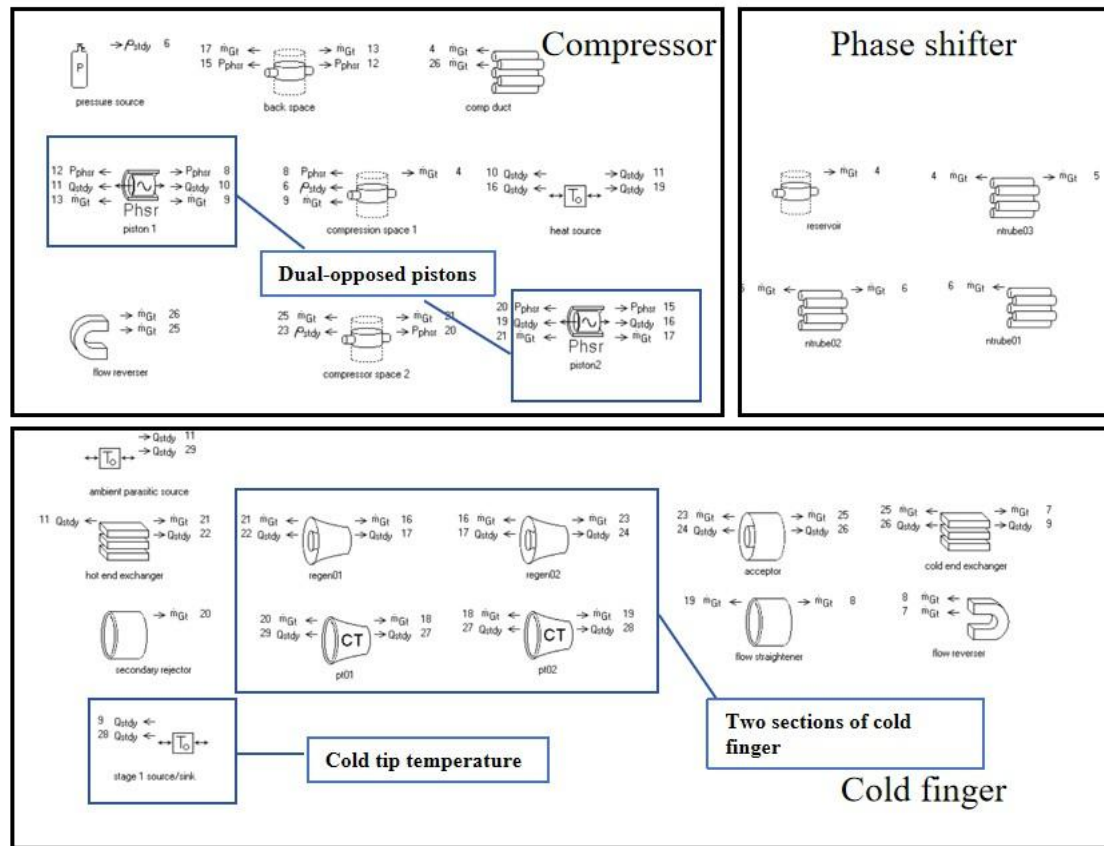


Figure 1. Sage simulation model of the PTC

3.2 Simulation results

Table 1 The different combinations of the inertance tubes

	Tube 1 length/m	Tube 2 length/m	Tube 3 length/m	Total length/m	Cooling capacity/W
Case 1	0.280	0.809	3.971	5.060	1.144
Case 2	0.020	0.607	1.314	1.941	1.143
Case 3	-	1.424	-	1.424	1.133

Table 1 shows the different combinations of the inertance tubes. The volume of the reservoir is constant. The diameters of the tube 1, tube 2, and tube 3 are 1 mm, 2 mm, and 3 mm. The cooling capacity is simulated based on the performance at an input power of 10 W and a cold tip temperature of 150 K. The operating frequency is 150 Hz.

Figure 2 illustrates the along-tube parameters, including the pressure amplitude and mass flow amplitude, and the phase between them in Case 1. The amplitude can be directly obtained from the model's calculation results, while the phase angle requires additional post-processing. The phase angle curve is not perfectly smooth, mainly due to two factors: the limited computational capacity of Sage, which restricts the number of spatial and temporal nodes, and the deviation of the waveform from a pure sine wave at some locations. Different colors represent tubes with different diameters. Position 0 indicates the inlet of the inertance tube, which corresponds to the hot end of the pulse tube. The outlet of the inertance tube is connected to the reservoir. The occurrence of peak values in the mass flow amplitude or pressure amplitude leads to a phase shift of nearly 180 degrees. The phase angle being less than -90 degrees is due to the distortion of the pressure wave near the extreme points at the inlet of the gas reservoir, causing a deviation from the sine wave and leading to a bias in the calculated phase. However, this does not impact the qualitative analysis. In the third tube with a diameter of 3 mm, the pressure amplitude, the mass flow amplitude, and the phase all exhibit periodic variations, which means that other tube lengths could also achieve a similar performance.

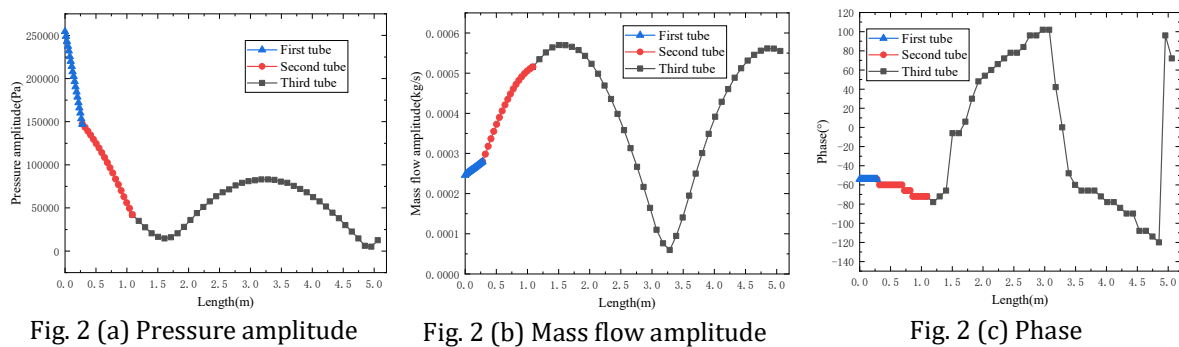


Figure 2. The along-tube parameters of Case 1.

After adjusting the length, Case 2 was obtained. The cooling capacity of the cryocooler remains virtually unchanged between Case 1 and Case 2, indicating that different combinations of the inertance tubes can achieve the same performance. Comparing the lengths of each tube, the length of the third tube has been reduced to one-third of its original length, while the second tube shows minimal variation. The length of the first tube has been significantly shortened to only 20 mm, suggesting that the model considers the first tube with a diameter of 1 mm to be unnecessary in this situation.

Similarly, the along-tube parameters of the inertance tubes in Case 2 were extracted, as shown in Fig. 3. The pressure amplitude and mass flow amplitude display a consistent trend along the tube length. To compare the values at the inlet of the inertance tube under the two different combinations, the pressure amplitudes are 254500 Pa and 256500 Pa, while the mass flow amplitudes are 0.2458 g/s and 0.2444 g/s, respectively. The phase angle between the pressure and mass flow is 54 degrees in both cases. These nearly identical parameters resulted in no significant change in the performance of the cryocooler.

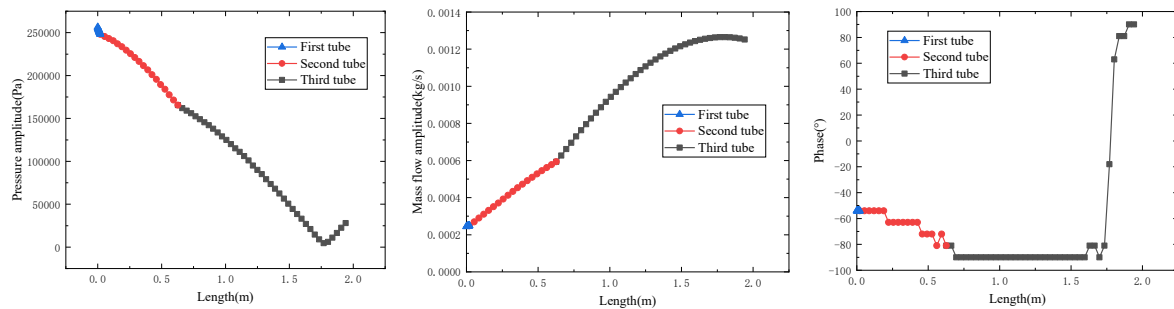


Fig. 3 (a) Pressure amplitude

Fig. 3 (b) Mass flow amplitude

Fig. 3 (c) Phase

Figure 3. The along-tube parameters of Case 2.

To compare the differences between a single inertance tube and a multi-diameter inertance tube combination, a single 2 mm tube was used for the calculation. The results, as shown in Case 3, indicate a length of 1.424 m and a cooling capacity of 1.133 W. The variation of along-tube parameters in Case 3 is similar to that in Case 2. However, the inlet mass flow amplitude of the inertance tube is slightly reduced, leading to a decrease in the mass flow within the regenerator, which results in a decline in cooling capacity. The specific inlet parameters for these three cases are presented in Table 2.

Table 2 The inlet parameters for the three cases.

	Pressure amplitude/Pa	Mass flow amplitude/g/s	Phase angle/°
Case 1	254500	0.2458	-54
Case 2	256500	0.2444	-54
Case 3	238000	0.2377	-54

4. Experimentation

4.1 The PTC prototype

The test system displayed in Figure 4 mainly consists of three parts: the compressor, cold finger, and phase shifter. A back-to-back moving coil linear compressor is used, connecting a coaxial cold finger via a tube. During testing, the cold head is situated in a vacuum environment below 10^{-4} Pa to minimize heat leakage. The cold tip temperature is measured using a PT100 platinum resistance thermometer. A resistor attached to the cold head is used to measure the cooling capacity. The reject temperature is controlled at 300 K by a water cooling unit. The inertance tubes and reservoir serve as the phase shifter, and the connections between the inertance tubes, pulse tube, and reservoir are all detachable, allowing for easy testing with various combinations.

4.2 Experimental results

Fig. 5 shows the performance of the cryocooler under different inertance tube combinations. Case 4 consists of three tubes, and case 5 uses a tube with a diameter of 2 mm. Table 3 presents the specific lengths of these two configurations and the cooling capacity of the cryocooler when the cold tip temperature is 150 K. The experimental results show that both the total length and

cooling capacity are in close agreement with the simulation results. These results indicate that using a single-diameter tube leads to a performance reduction. However, if smaller size and lighter weight are prioritized, adopting a single inertance tube can make the cryocooler more lightweight, albeit with a slight sacrifice in performance.



Figure 4. The pulse tube cryocooler prototype.

Table 3 The experimental and simulation results of different inertance tube combinations

Experimental results				Simulation results		
	Case	Total length/m	Cooling capacity/W	Case	Total length/m	Cooling capacity/W
Multi-diameter	Case 4	1.8	1.13	Case 2	1.9	1.14
Single-diameter	Case 5	1.3	1.06	Case 3	1.4	1.13

5. Conclusion

The impact of single-stage and multi-diameter inertance tubes on the performance of the cryocooler has been numerically and experimentally investigated in this paper. Based on the Sage software, the along-tube parameters for various combinations of inertance tubes were analyzed. The results demonstrate that combinations of inertance tubes with different lengths and diameters can achieve the same inlet conditions, thereby resulting in identical performance of the cryocooler. Guided by these analyses, the experiments were conducted, and the experimental data showed good consistency with the simulation results. The results indicate that, for micro pulse tube cryocoolers, using a single-stage inertance tube can significantly reduce the weight and size

of the cryocooler with minimal performance compromise, thereby facilitating the lightweight design of the cryocoolers.

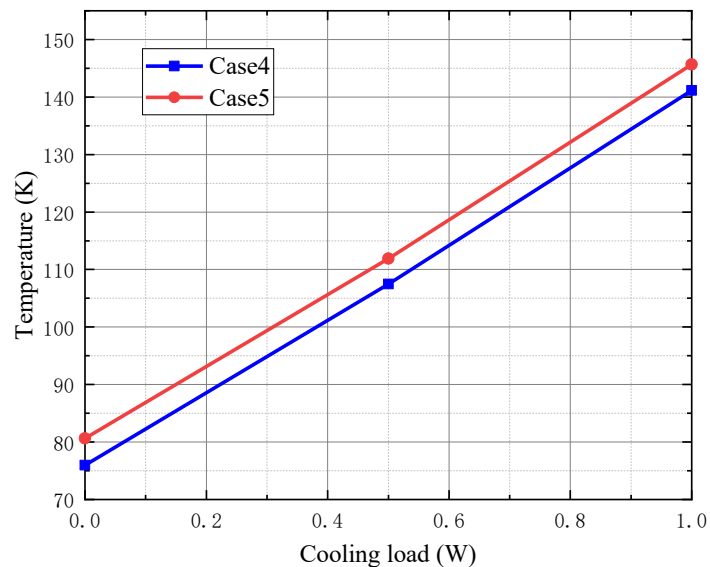


Figure 5. The cryocooler performance under different inertance tube combinations.

Acknowledgments

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